1.4 Notes and Examples

Name: Block: Seat:

Continuity and the Intermediate Value Theorem (IVT)

Definition of Continuity at a Point:
A function f is said to be ____________________ at $x = c$ if and only if:

1.

2.

3.

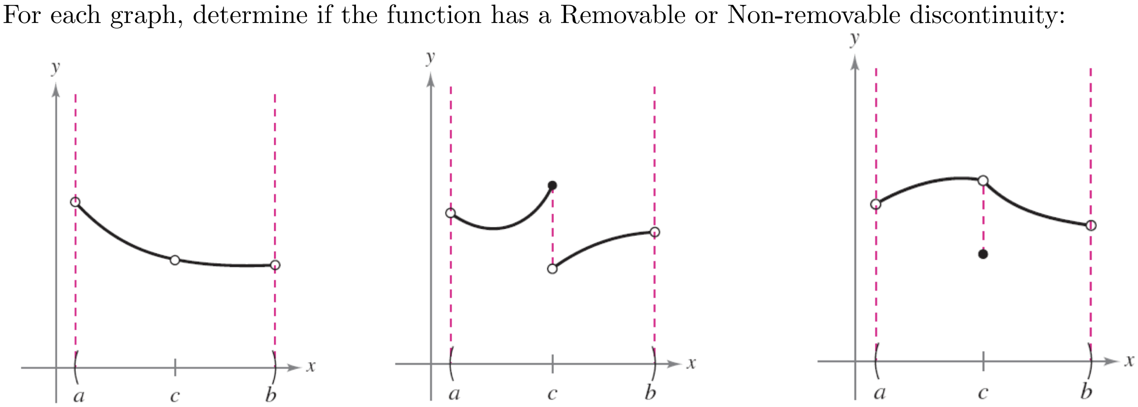
1. Sketch a graph of a function that demonstrates the following:

   (a) $f(c)$ is not defined

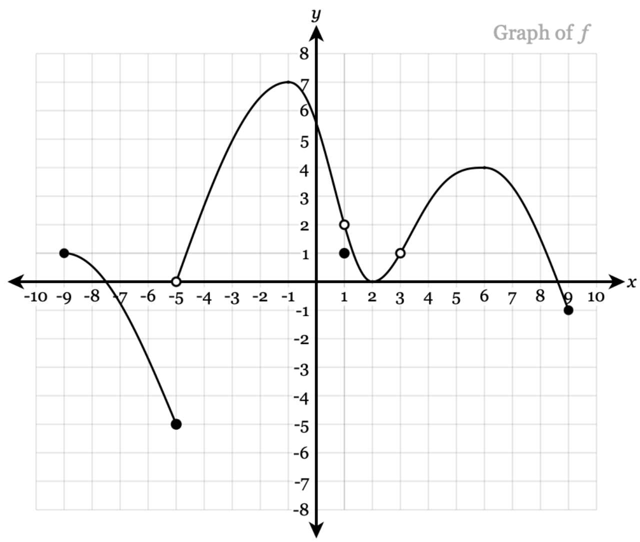
   (b) $\lim\limits_{x \to c} f(x)$ DNE

   (c) $f(c)$ <u>IS defined</u> and $\lim\limits_{x \to c} f(x)$ <u>DOES exist</u>, but $\boxed{f(c) \neq \lim\limits_{x \to c} f(x)}$

2. For each graph, determine if the function has a Removable or Non-removable discontinuity:

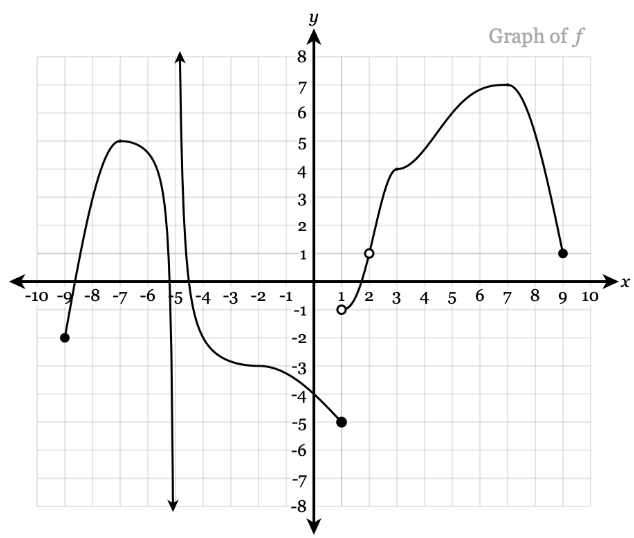


`http://webspace.ship.edu/msrenault/GeoGebraCalculus/continuity_at_a_point.html`



3. State all the values of x in the open interval $(-9, 9)$ has:
   (a) removable discontinuities:

   (b) non-removable discontinuities:



4. State all the values of x in the open interval $(-9, 9)$ has:
   (a) removable discontinuities:

   (b) non-removable discontinuities:

see DeltaMath Lab "Types of Discontinuities (Graphically)" , and "Demonstrating Continuity from a Graph".

5. Let $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0 \end{cases}$. Show that f is continuous at $x = 0$

   (a)

   (b)

   (c)

6. Let $f(x) = \begin{cases} -2x + 2 & \text{for } x \leq 0 \\ x^2 + 1 & \text{for } x > 0 \end{cases}$. Is the function f continuous at $x = 0$? Justify your answer.

   (a)

   (b)

   (c)

7. Let $f(x) = \begin{cases} x^2 + kx & \text{if } x > 2 \\ 8x - k & \text{if } x \leq 2 \end{cases}$.

   Use the definition of continuity to find the value of k that will make the function continuous everywhere.

see DeltaMath Lab "Continuity - Find k", and "Continuity"

The IVT (Intermediate Value Theorem)

3 Conditions: IF

1. f is ____________________ on the closed interval $[a, b]$,

2. $f(a)$_________$f(b)$

3. And there is a k where _________ $\leq k \leq$ _________,

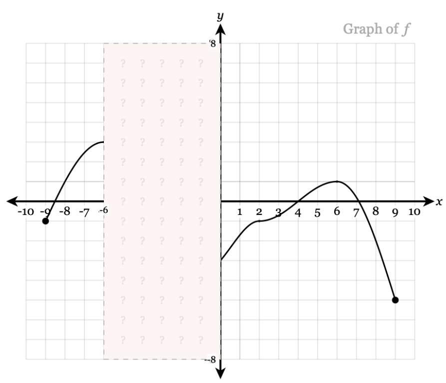
THEN there is at least one number c in $[a, b]$ such that $f(c) = k$

In other words, if <u>and only if</u> a function is ____________________ on a ________________ interval $[a, b]$,

f takes on ___________ value between $f(a)$ and _____________. Further, for any value k on $[f(a), f(b)]$,

there MUST BE one or more numbers c on the interval _____________where $f(c) =$ ______________

8. The function $f(x)$ is continuous on its domain of $[-9, 9]$ and is plotted below such that the portion of the graph on the interval $(-6, 0)$ is hidden from view. Given that $f(-6) = 3$ and $f(0) = -3$, is there a value c on the interval $(-6, 0)$ where $f(c) = 0$?.



(a) Check if Conditions are met (Can the IVT be applied? Why?):

(b) Next name the Theorem, and state the conclusion in context of <u>this f</u>

1. "by the ______________, since ______________ $\leq 0 \leq$ ______________,

2. there exists a value c where ______________ $\leq c \leq$ ______________

3. such that $f(c) =$______________.

4. "Hence, there is a value of c on the interval ______________ where $f(c) =$ ______________

see DeltaMath Lab "Intermediate Value Theorem" to practice this

Some Theorems That Can Be Used To Justify Continuity
If f and g are known to be continuous functions :

1. ________________ of continuous functions are continuous (i.e. if $b \in \mathbb{R}$, $b * f(x)$ is continuous)
2. ______________________ of continuous functions are continuous (i.e .$f(x) \pm g(x)$ is continuous)
3. __________________________ of continuous functions are continuous (i.e. $f(x)g(x)$ is continuous)
4. ____________________ of continuous functions are continuous (i.e. if $g(x) \neq 0$, $\dfrac{f(x)}{g(x)}$ is continuous)
5. _______________________ of continuous functions are continuous (i.e. $f(g(x))$ is continuous)
6. _______________________ functions are continuous (i.e. $y = x^n$ where n is an integer)
7. Sinusoidal functions are continuous (e.g. $y =$ _____________ or $y =$_____________)
8. Next chapter, we will discover that some functions can be called "differentiable." There is a later theorem worth mentioning here: __________________________ functions must be continuous.

9. Given $f(x) = x^2 + 5x - 6$, are there one or more values of c, $-1 \leq c \leq 2$ where $f(c) = 4$? Justify.
Hint: Remember to check what $f(a)$ and $f(b)$ are...

10. Given $f(x) = x^2 + 3x - 1$, are there one or more values of c, $-1 \leq c \leq 2$ where $f(c) = 0$? Justify.

11. Given $f(x) = \dfrac{1}{x-2}$, are there one or more values of c, $\dfrac{5}{7} \leq c \leq 7$ where $f(c) = \dfrac{1}{4}$? Justify.

Some AP Style Questions

12. Let $f(x) = x^3 + 2x - 1$. Show that f must have one or more zeros in the closed interval $[0, 1]$

13. Functions g and h are differentiable with $g(2) = h(2) = 4$. It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying

$$g(x) \leq k(x) \leq h(x)$$

for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

Hint: They say, "If the only thing in your hand is a hammer, everything starts looking like a nail."
Section 1.4 has more than one big idea. Ask yourself, "Self, is this a job for the IVT, or the definition of Continuity?"

14. A certain train runs back and forth on an East-West section of railroad track. Its velocity, measured in meters per minute, is given by a differentiable function $v(t)$, where t is measured in minutes. Selected values for $v(t)$ are given in the table below:

t (minutes)	0	2	5	8	12
$v(t)$ in (meters/minute)	0	100	40	-120	-150

Do the data in the table support the conclusion that the train's velocity is -100 meters per minute at some time t between $t = 5$ minutes and $t = 8$ minutes? Justify your answer.

15. The table below shows selected values of a continuous function g. For $0 \leq x \leq 11$, what is the fewest possible number of times that $g(x) = 2$? Justify your answer.

x	0	2	5	9	11
$g(x)$	1	2.8	1.7	1	3.4